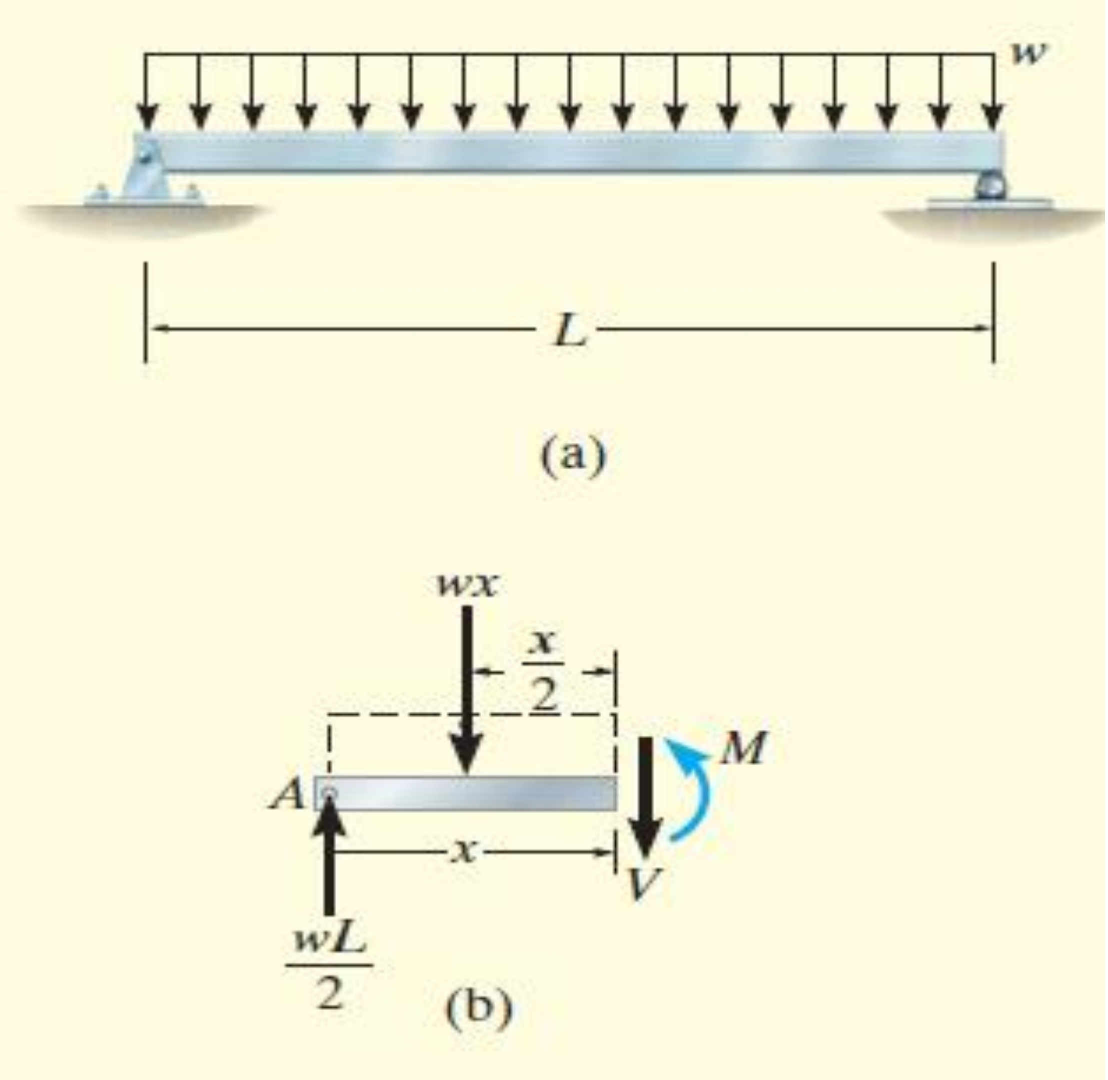
Example 1 Draw the shear and moment diagrams for the beam shown in Fig. 6–4a.



$$+\uparrow\Sigma F_y = 0; \qquad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \qquad (1)$$

$$\circlearrowleft + \Sigma M = 0; \qquad -\left(\frac{wL}{2}\right)x + (wx)\left(\frac{x}{2}\right) + M = 0$$

$$M = \frac{w}{2}(Lx - x^2) \qquad (2)$$

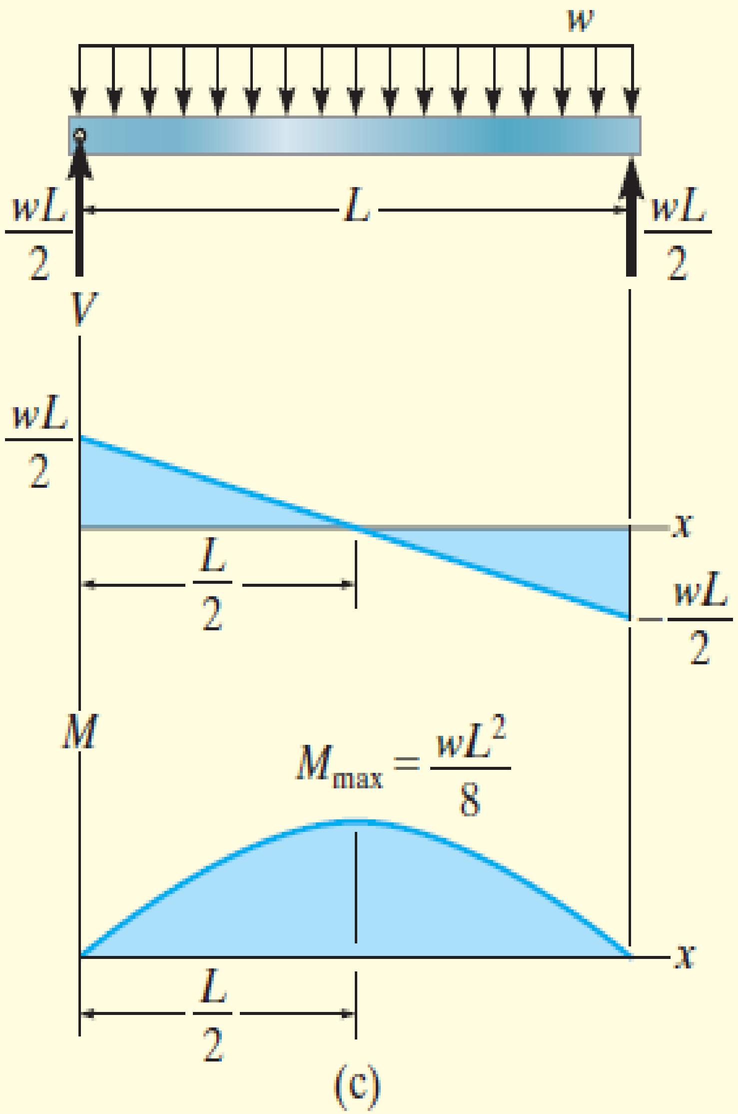


Fig. 6–4

Shear and Moment Diagrams. The shear and moment diagrams shown in Fig. 6–4*c* are obtained by plotting Eqs. 1 and 2. The point of *zero shear* can be found from Eq. 1:

$$V = w\left(\frac{L}{2} - x\right) = 0$$

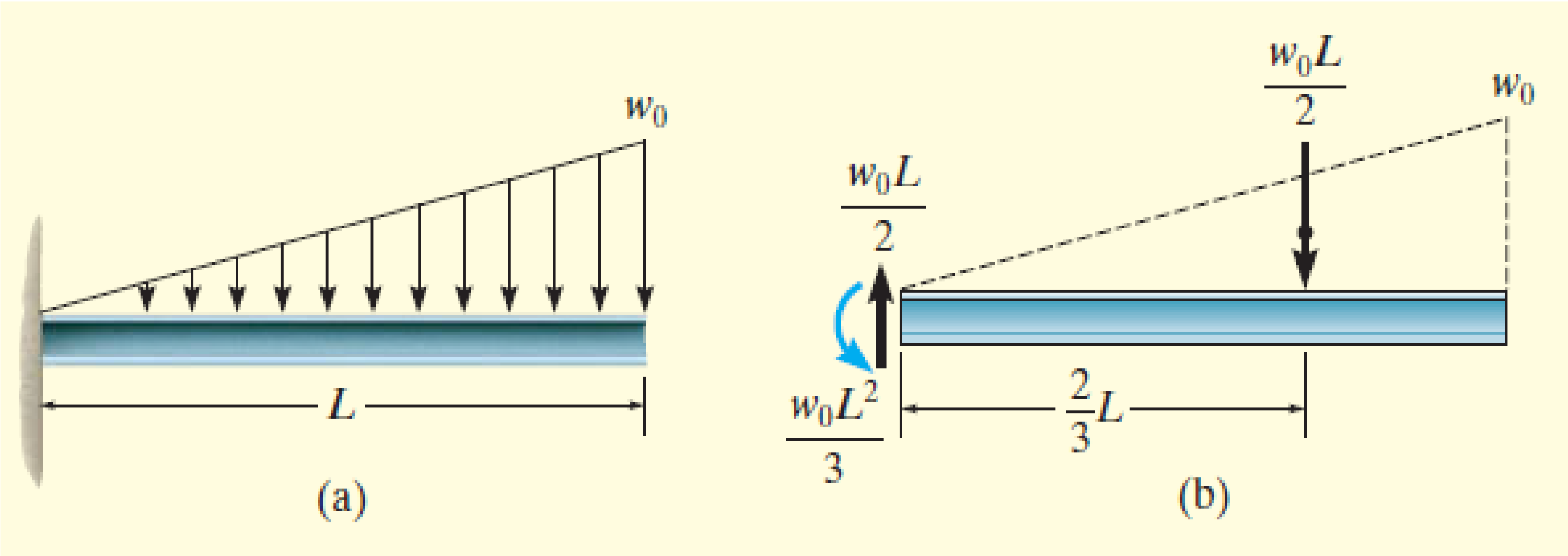
$$x = \frac{L}{2}$$

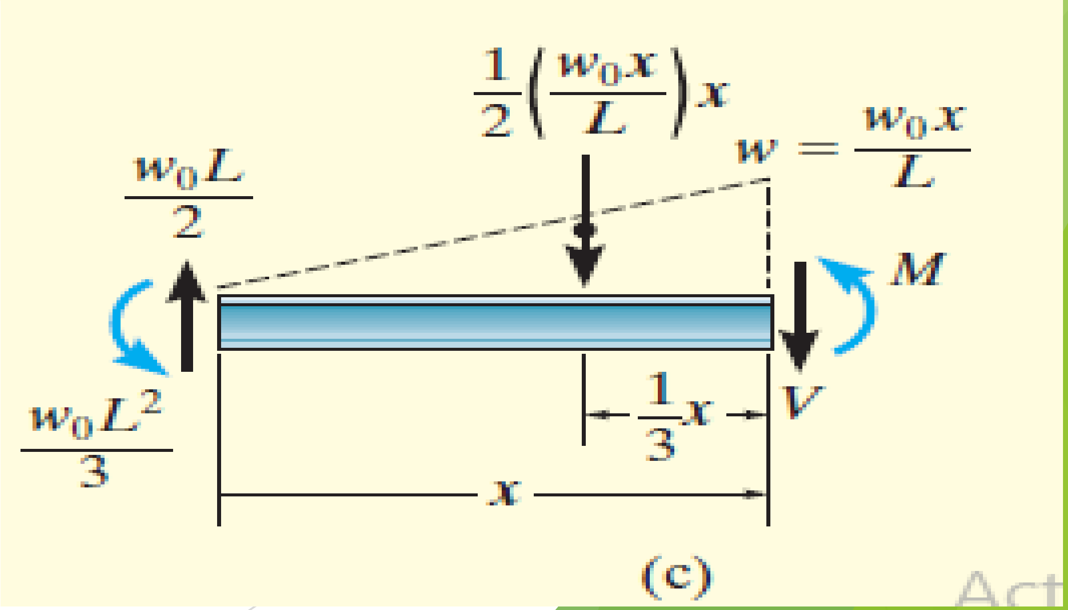
NOTE: From the moment diagram, this value of x represents the point on the beam where the *maximum moment* occurs, since by Eq. 6–2 (see Sec. 6.2) the *slope* $V = dM/dx = 0$. From Eq. 2, we have

$$M_{\max} = \frac{w}{2}\left[L\left(\frac{L}{2}\right) - \left(\frac{L}{2}\right)^2\right]$$

$$= \frac{wL^2}{8}$$

Example 2: Draw the shear and moment diagrams for the beam shown in Fig. 6–5a.





$$+\uparrow \Sigma F_y = 0; \qquad \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x - V = 0$$

$$V = \frac{w_0}{2L}(L^2 - x^2) \tag{1}$$

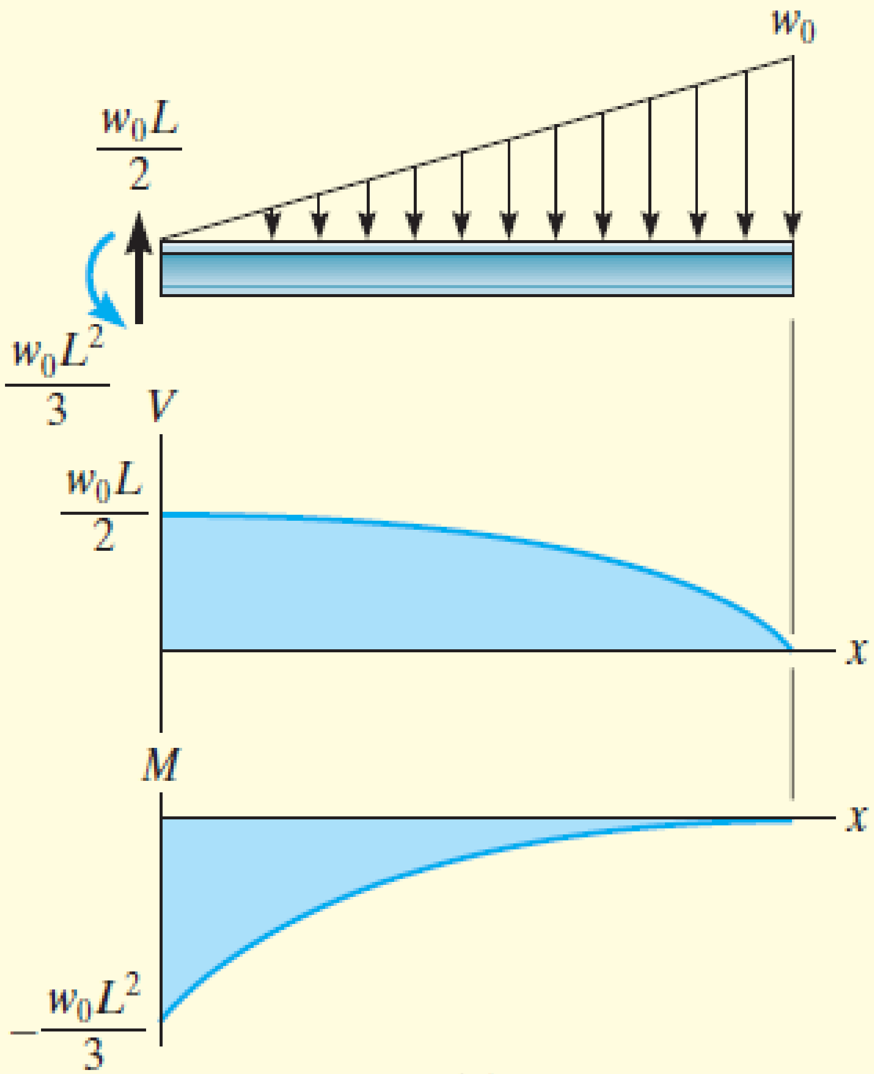
$$\circlearrowright + \Sigma M = 0; \qquad \frac{w_0 L^2}{3} - \frac{w_0 L}{2}(x) + \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right) + M = 0$$

$$M = \frac{w_0}{6L}(-2L^3 + 3L^2 x - x^3) \tag{2}$$

These results can be checked by applying Eqs. 6–1 and 6–2 of Sec. 6.2, that is,

$$w = \frac{dV}{dx} = \frac{w_0}{2L}(0 - 2x) = -\frac{w_0 x}{L} \qquad \text{OK}$$

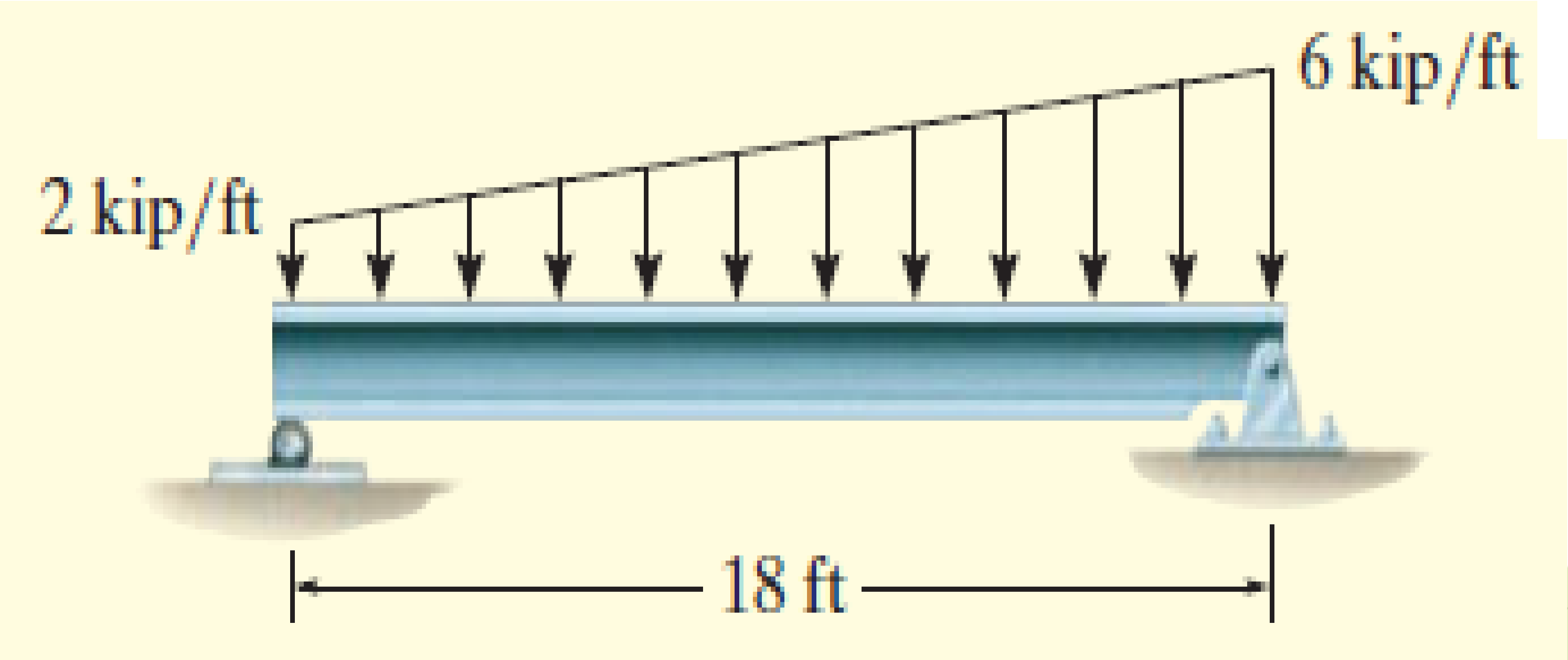
$$V = \frac{dM}{dx} = \frac{w_0}{6L}(0 + 3L^2 - 3x^2) = \frac{w_0}{2L}(L^2 - x^2) \qquad \text{OK}$$



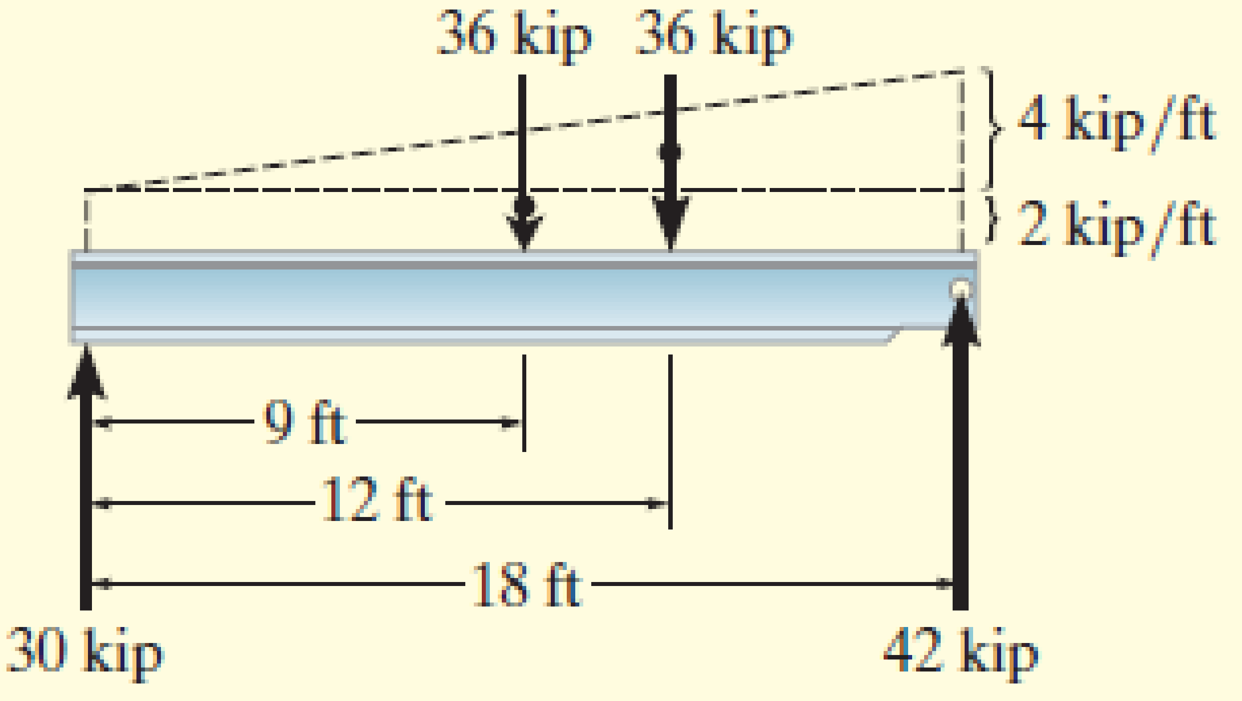
(d)

EXAMPLE 3

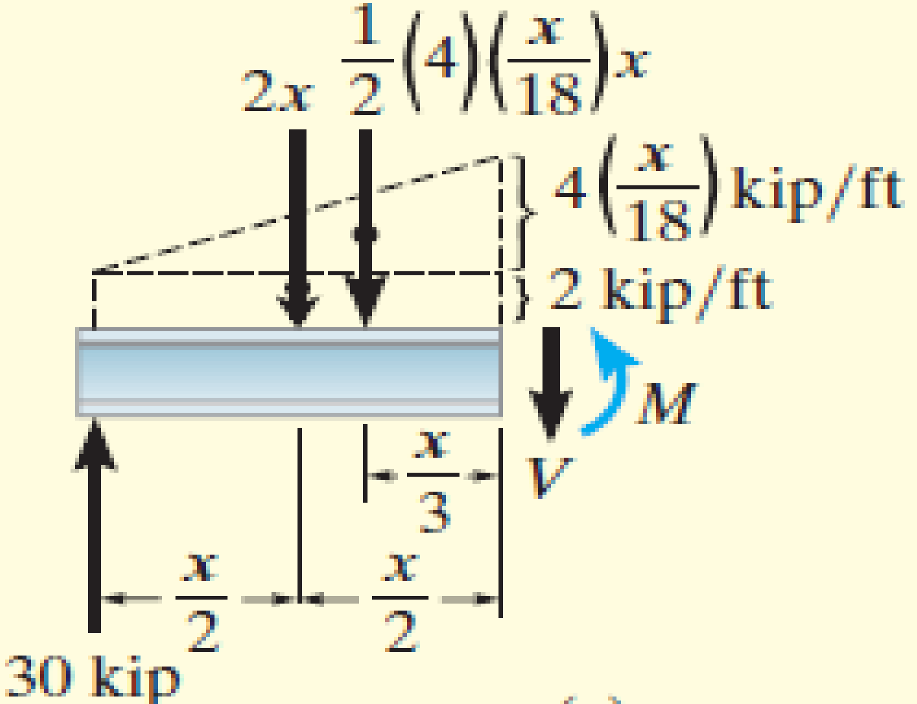
Draw the shear and moment diagrams for the beam shown in Figure.



(a)



(b)



(c)

$$+\uparrow \Sigma F_y = 0;\; 30 \text{ kip} - (2 \text{ kip/ft})x - \frac{1}{2}(4 \text{ kip/ft})\left(\frac{x}{18 \text{ ft}}\right)x - V = 0$$

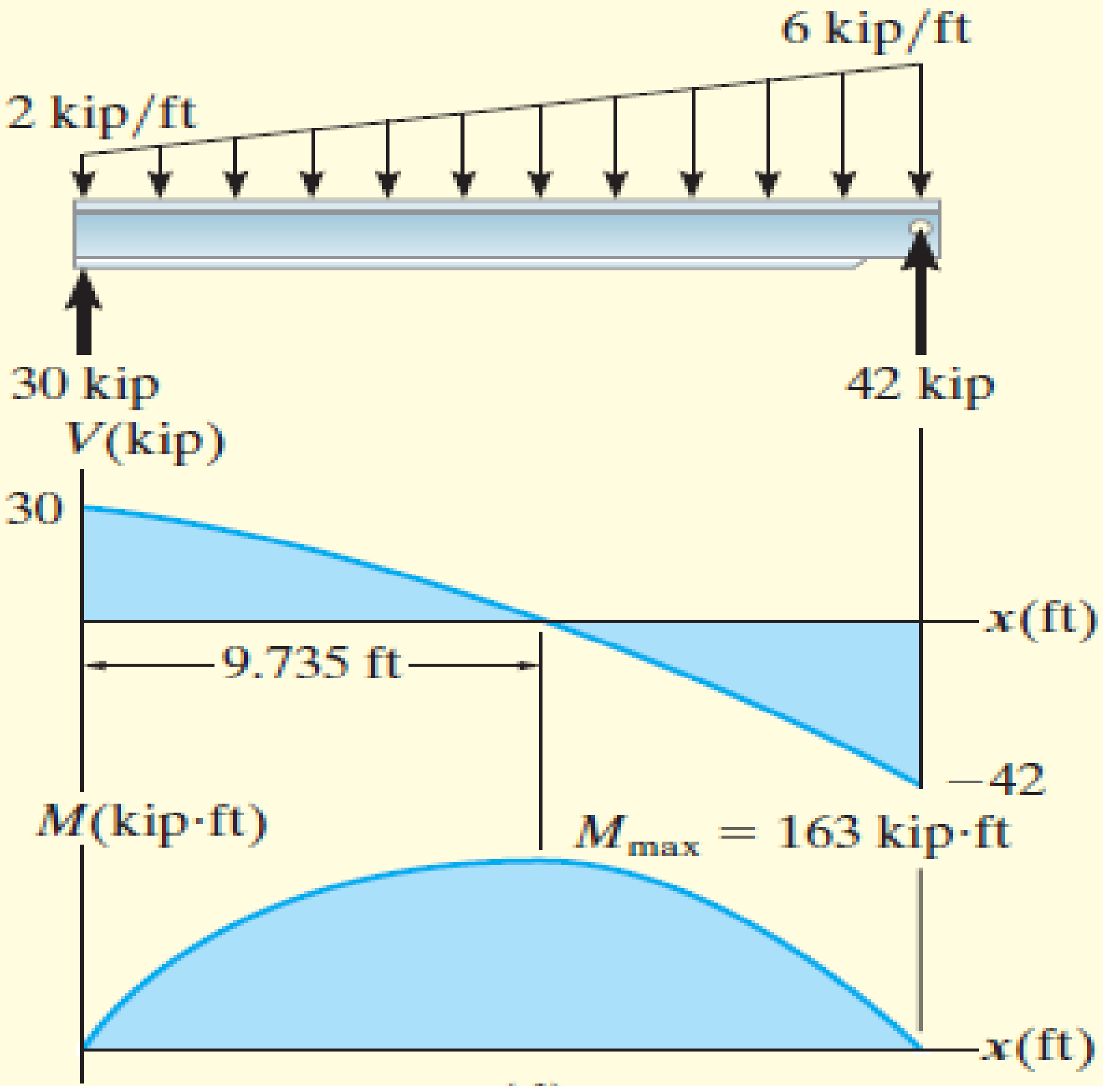
$$V = \left(30 - 2x - \frac{x^2}{9}\right) \text{kip} \qquad (1)$$

$$\curvearrowright + \Sigma M = 0;$$

$$-30\ \text{kip}(x) + (2\ \text{kip/ft})x\left(\frac{x}{2}\right) + \frac{1}{2}(4\ \text{kip/ft})\left(\frac{x}{18\ \text{ft}}\right)x\left(\frac{x}{3}\right) + M = 0$$

$$M = \left(30x - x^2 - \frac{x^3}{27}\right)\text{kip}\cdot\text{ft} \qquad (2)$$

Equation 2 may be checked by noting that $dM/dx = V$, that is, Eq. 1. Also, $w = dV/dx = -2 - \frac{2}{9}x$. This equation checks, since when $x = 0$, $w = -2$ kip/ft, and when $x = 18$ ft, $w = -6$ kip/ft, Fig. 6–6a.



(d)

Shear and Moment Diagrams. Equations 1 and 2 are plotted in Fig. 6–6*d*. Since the point of maximum moment occurs when $dM/dx = V = 0$ (Eq. 6–2), then, from Eq. 1,

$$V = 0 = 30 - 2x - \frac{x^2}{9}$$

Choosing the positive root,

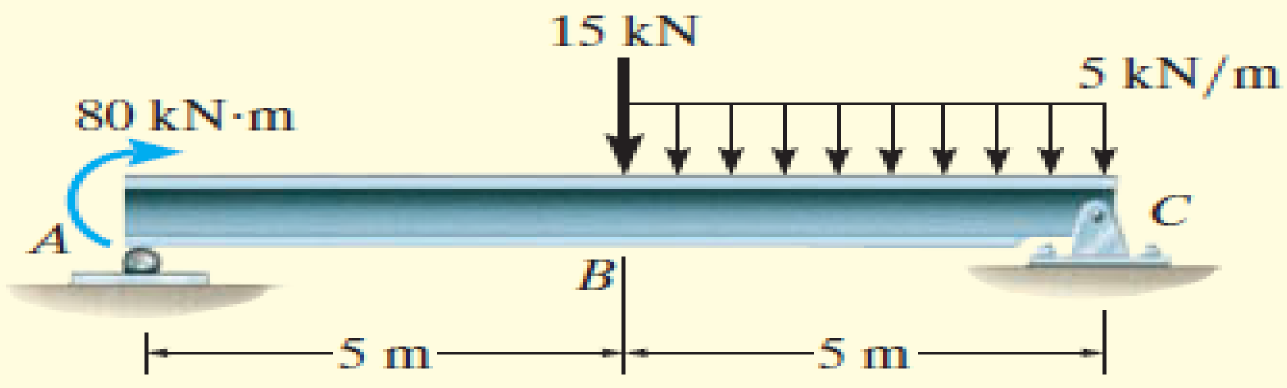
$$x = 9.735 \text{ ft}$$

Thus, from Eq. 2,

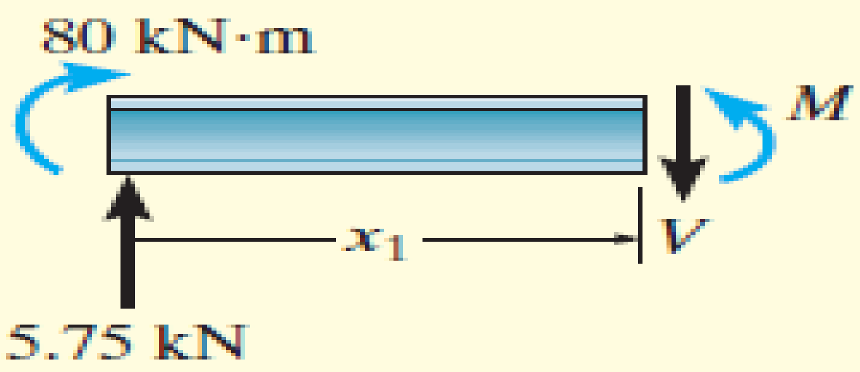
$$M_{max} = 30(9.735) - (9.735)^2 - \frac{(9.735)^3}{27}$$
$$= 163 \text{ kip} \cdot \text{ft}$$

EXAMPLE 4

Draw the shear and moment diagrams for the beam shown in Figure.



(a)



(b)

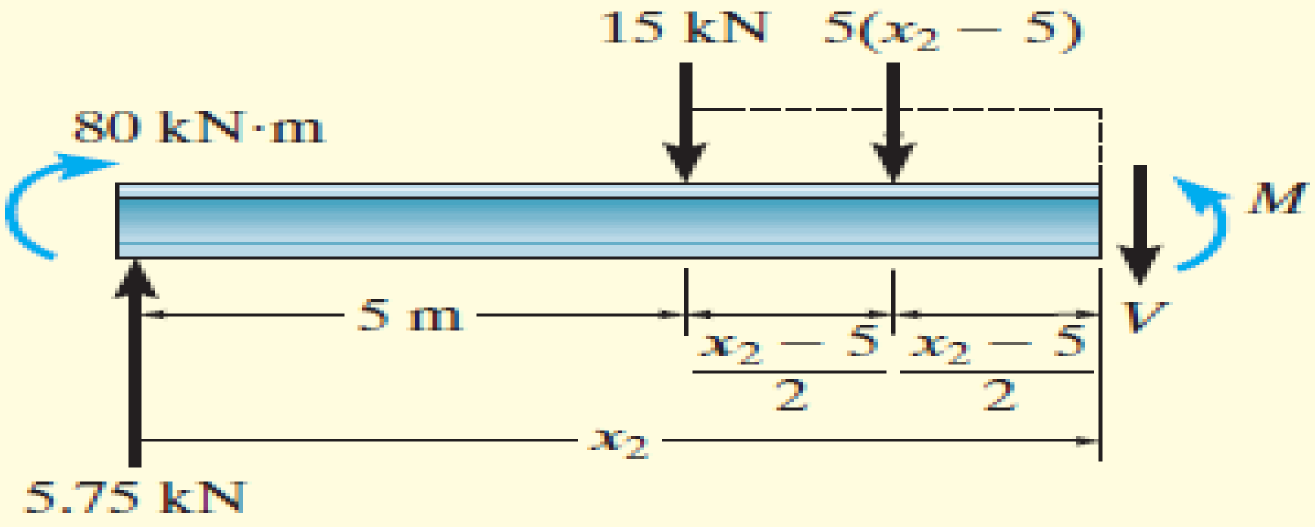
$0 \leq x_1 < 5$ m, Fig. 6–7b:

$$+\uparrow \Sigma F_y = 0; \qquad 5.75 \text{ kN} - V = 0$$

$$V = 5.75 \text{ kN}$$

$$\circlearrowright + \Sigma M = 0; \qquad -80 \text{ kN}\cdot\text{m} - 5.75 \text{ kN}\, x_1 + M = 0$$

$$M = (5.75x_1 + 80) \text{ kN}\cdot\text{m}$$



(c)

$5\text{ m} < x_2 \leq 10\text{ m}$, Fig. 6–7*c*:

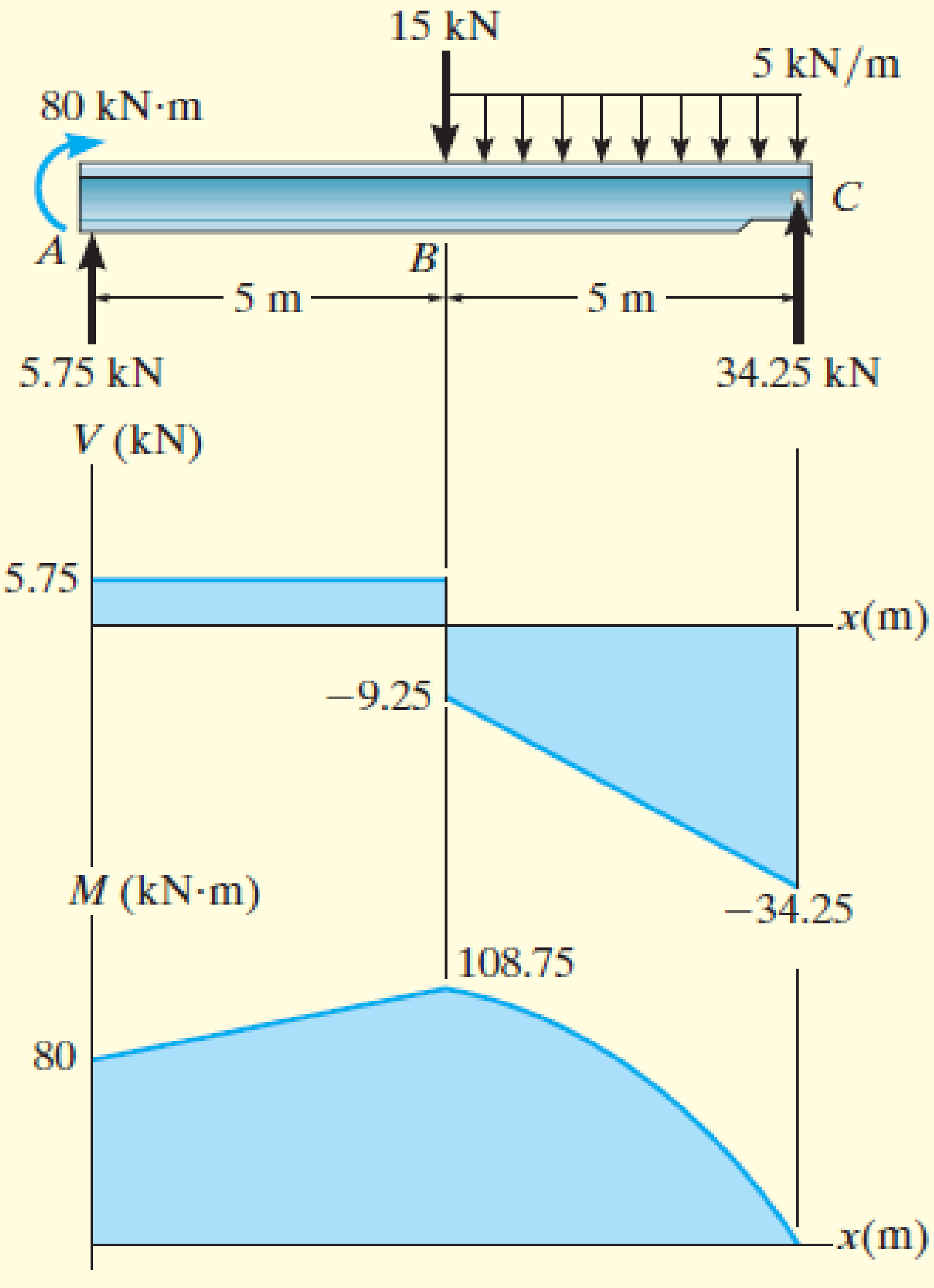
$$+\uparrow \Sigma F_y = 0; \quad 5.75\text{ kN} - 15\text{ kN} - 5\text{ kN/m}(x_2 - 5\text{ m}) - V = 0$$

$$V = (15.75 - 5x_2)\text{ kN} \qquad (3)$$

$$\circlearrowright\!+ \Sigma M = 0; \quad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\,x_2 + 15\text{ kN}(x_2 - 5\text{ m}) + 5\text{ kN/m}(x_2 - 5\text{ m})\left(\frac{x_2 - 5\text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2^2 + 15.75x_2 + 92.5)\text{ kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that $w = dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN·m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–7*d*.



(d)